APPENDIX C

CHANNEL STABILITY ANALYSIS

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C.1. Introduction. The stability analysis for Bogue Inlet reproduces a similar analysis conducted for Oregon Inlet by Mr. Bill Dennis, Coastal Engineer with the Wilmington District U.S. Army Corps of Engineers. Mr. Dennis's analysis for Oregon Inlet was published in the Phase II, Supplement No. 3, General Design Memorandum for the Manteo (Shallowbag) Bay Project, North Carolina (USACE 1998). The following discussion of the analytical method closely follows and/or paraphrases Mr. Dennis's presentation. Note that the analytical procedure is also described in the Corps of Engineers Coastal Engineering Manual (USACE 2002).

C.2. Wave generated and other currents along the coast move sand into the inlet channel reducing its cross-sectional area. Inlet flow will tend to scour any deposition, which reduces the cross-section below its equilibrium size. Escoffier (1940, 1977) developed a method to analyze the stability of tidal inlets by combining hydraulic stability criteria with sedimentary stability. The hydraulic stability is based on the work of Keulegan (1967) who solved the equations of motion governing flow through an inlet connecting the ocean with an enclosed bay. The results of Keulegan's analysis, expressed in terms of a dimensionless parameter K know as the repletion coefficient, is as follows:

$$K = (T/2\pi a_o)\ (A_c/A_B)\ (2ga_o/F)^{1/2}$$

Where:

- T = Tidal period (44,640 seconds)
- a_o = ocean tidal amplitude (2.15 feet for spring tide)
- A_c = Inlet cross-sectional area (square feet below MSL)
- A_B = Bay surface area (square feet)
- g = acceleration of gravity (32.2 fps^2)
- F = Inlet impedance = $k_{en} + k_{ex} + (2gn^2L_c/h_c^{4/3})$
- k_{en} = entrance loss coefficient = 0.1
- k_{ex} = exit loss coefficient = 1.0
- n = Manning's n = 0.025
- h_c = mean channel depth
- L_c = effective channel length = 6,000 feet

The relationship between K and the ratio of bay to ocean tidal amplitude (a_B/a_o) and the tidal phase lag (ε) for an inlet bay system is shown on Figure C.1. Keulegan also developed a relationship between K and the dimensionless maximum velocity (V_{max}') with this relationship shown on Figure C.2. The maximum velocity through an inlet is related to V_{max}' as follows:

$$Vmax = (V_{max}')\ (2\pi a_o/T)\ (A_B/A_c)$$

C.3. These two relationships are used to generate a stability curve for known inlet characteristics. A generalized stability curve is shown on Figure C.3, which shows the relationship between the maximum velocity in the inlet to the channel cross-sectional

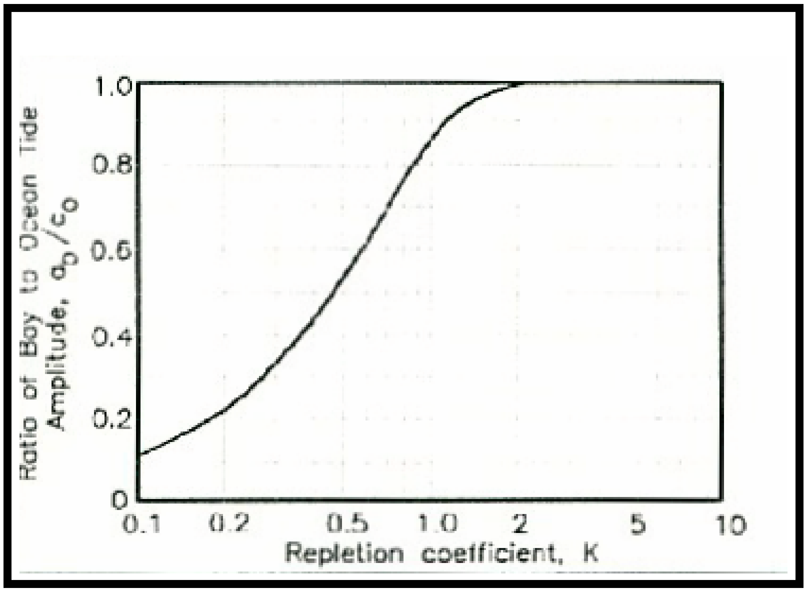

Figure C.1 K versus a_B/a_O

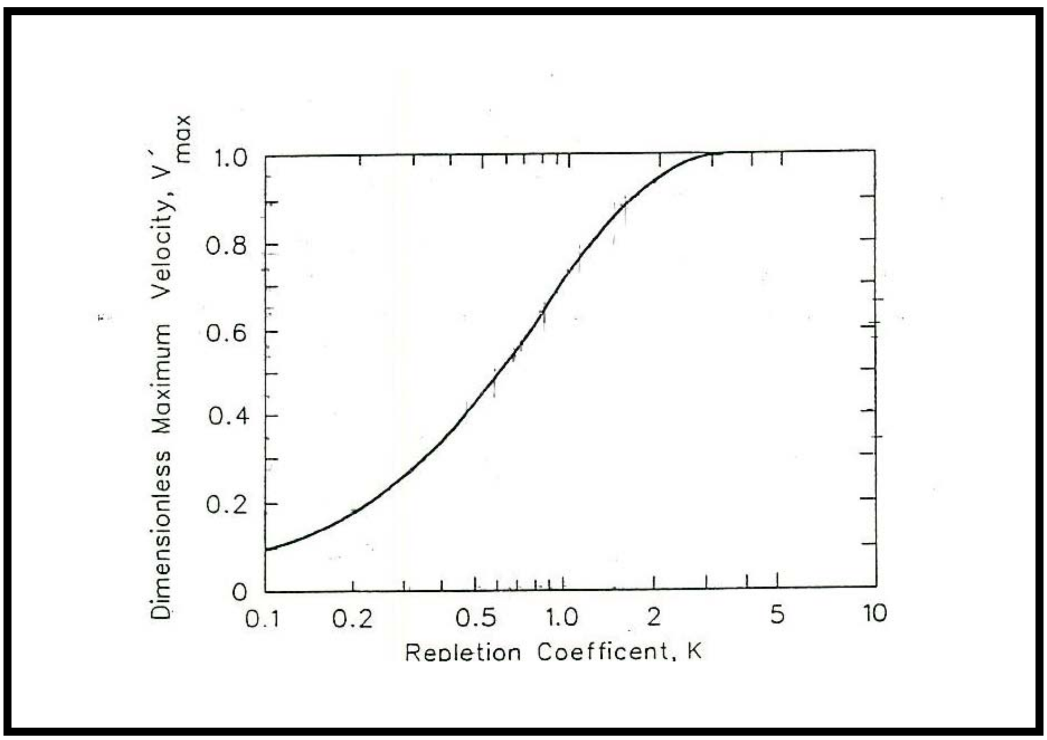

Figure C.2 K versus Dimensionless Maximum Velocity (V_{Mzx})

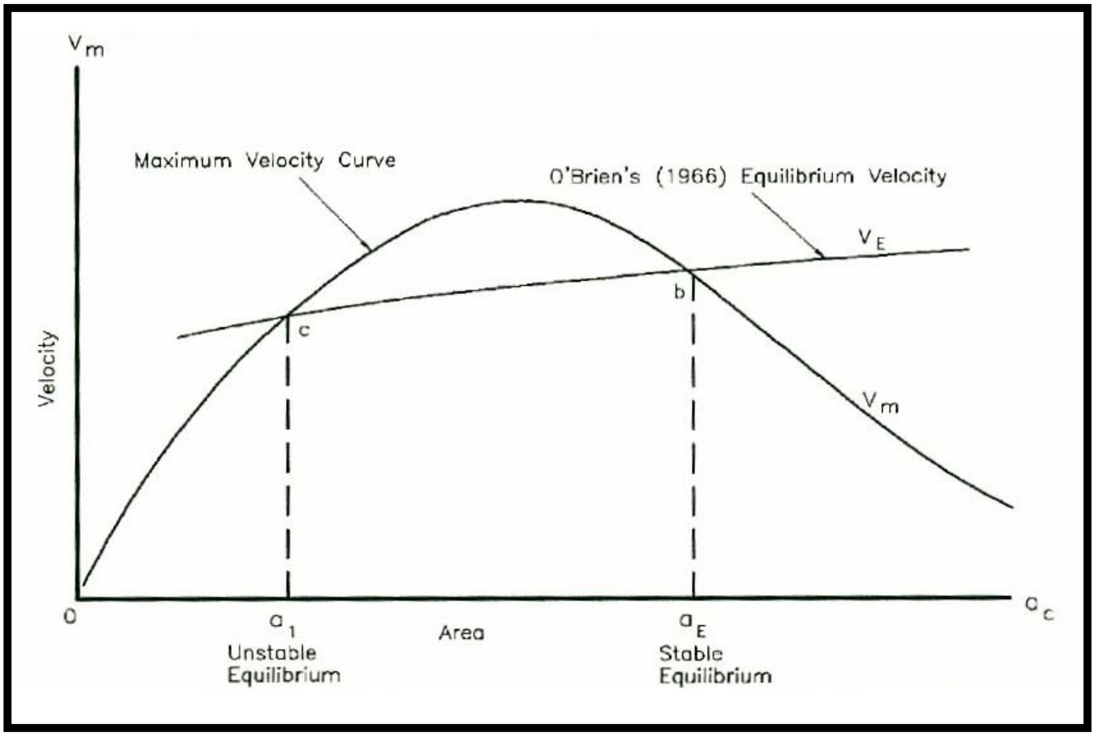

Figure C.3 Generalized Stability Curve

area. As shown on Figure C.3, a peak velocity V_{max} occurs at a critical area A_c^*. Inlet cross-sectional areas greater than A_c^* (i.e. to the right of the peak velocity) are hydraulically stable whereas those less than A_c^* are hydraulically unstable. For a cross-sectional area in the unstable region, any induced change in the cross-section will result in additional reductions in the inlet's cross-sectional area. For example, if the cross-sectional area is reduced by sediment deposition, the velocity will reduce as will the ability of the inlet to scour the deposited sediment. This will further reduce the cross-sectional area and velocity and the inlet will continue to diminish in size until it closes. In the stable region of the curve (i.e. to the left of the peak velocity), changes in the cross-sectional area of the inlet will produce velocity changes that will tend to restore the cross-section back to the original area.

C.4. The stability curve for Bogue Inlet was developed by computing the repletion coefficient for a wide range of channel cross-sectional areas. The repletion coefficient depends on the surface area of the bay (A_B). For the Bogue Inlet system, the bay area is not readily defined; therefore an effective bay area was determined from the tidal prism of the inlet and the ratio of the bay tidal amplitude (a_B) to the ocean tidal amplitude (a_o). Based on published tide data, the ratio a_B/a_o for Bogue Inlet is approximately 0.33. For spring tide conditions, the amplitude of the bay tide would be 0.71 feet. Since the primary focus of the stability analysis was on the existing ebb channel not the entire inlet, only that portion of the tidal prism that flows through the existing channel was used to determine the effective bay area. The October 2001 flow measurements in Bogue Inlet conducted by CSE (CSE January 2002) indicated that 61.6% of the ebb tidal prism passes through the existing channel, accordingly, the ebb tidal prism associated with the existing channel is 4.72 x 10^8 cubic feet (= 61.6% x 7.66 x 10^8 cubic feet). The relationship between the tidal prism and the effective area of the bay ($A_{B(eff)}$) is given by:

$$P = 2a_B A_{B(eff)}$$

The effective area of the bay associated with the flow through the existing channel was determined from the above relationship to be 3.33 x 10^8 square feet.

C.5. An empirical relationship between the width of the channel and its depth, developed by Graham and Mehta 1981, was used to define the relationship between the channel area and its geometric properties. The empirical relationship is given by:

$$A_c = h_c W_c$$

and;

$$h_c = pW_c^q$$

in which the empirical coefficients p and q determined by Graham and Mehta are equal to 1.164 (metric units) and 0.19 (metric units) respectively.

C.6. Computations for the hydraulic stability curve relating V_{max} to A_c are provided in Table C.1 with the resulting curve plotted on Figure C.4. While the analysis was

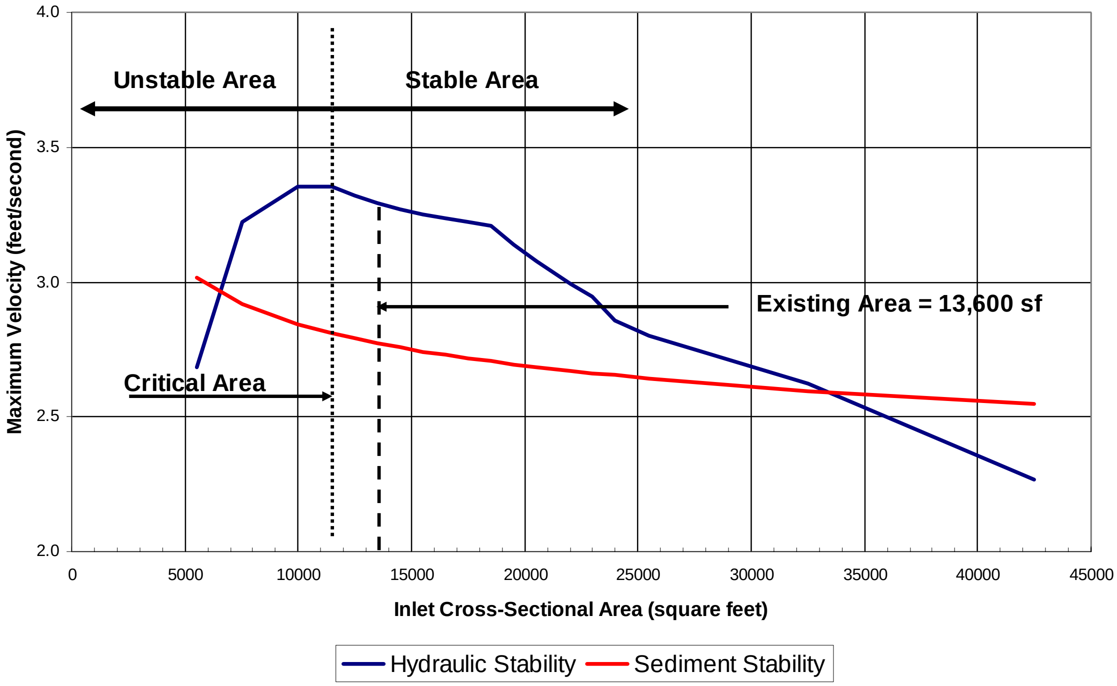

Figure C.4 Stability Analysis – Bogue Inlet

Table C.1
Bogue Inlet Stability Analysis

Cross-Sectional Area of Channel A_c	Total Inlet Cross-Sectional A_T	Channel Width W_c (feet)	Mean Channel Depth h_c (feet)	Hydraulic Criteria: Inlet Impedance F	Repletion Coefficient K	a_b/a_o	a_b	Channel Tidal Prism P (cf)	Dimensionless Velocity $V_{max'}$	Max Velocity V_{max} (fps)	Sedimentary Criteria: Equilibrium Tidal Prism P_e (cf)[(a)]	V_{max}[(b)]
3,000	5,500	327	9.2	13.69	0.09	0.110	0.24	1.57E+08	0.080	2.68	1.50E+08	3.02
5,000	7,500	502	10.0	12.39	0.17	0.190	0.41	2.72E+08	0.160	3.22	2.41E+08	2.92
7,500	10,000	706	10.6	11.45	0.26	0.290	0.62	4.15E+08	0.250	3.36	3.52E+08	2.84
9,000	11,500	823	10.9	11.06	0.32	0.390	0.84	5.58E+08	0.300	3.36	4.18E+08	2.81
10,000	12,500	899	11.1	10.84	0.36	0.420	0.90	6.01E+08	0.330	3.32	4.61E+08	2.79
11,000	13,500	974	11.3	10.64	0.39	0.440	0.95	6.29E+08	0.360	3.29	5.04E+08	2.77
12,000	14,500	1,048	11.5	10.46	0.43	0.470	1.01	6.72E+08	0.390	3.27	5.47E+08	2.76
13,000	15,500	1,121	11.6	10.31	0.47	0.500	1.08	7.15E+08	0.420	3.25	5.89E+08	2.74
14,000	16,500	1,193	11.7	10.16	0.51	0.560	1.20	8.01E+08	0.450	3.24	6.31E+08	2.73
15,000	17,500	1,264	11.9	10.03	0.55	0.590	1.27	8.44E+08	0.480	3.22	6.73E+08	2.72
16,000	18,500	1,334	12.0	9.91	0.59	0.620	1.33	8.87E+08	0.510	3.21	7.15E+08	2.71
17,000	19,500	1,404	12.1	9.79	0.64	0.650	1.40	9.30E+08	0.530	3.14	7.57E+08	2.69
18,000	20,500	1,473	12.2	9.69	0.68	0.670	1.44	9.58E+08	0.550	3.08	7.98E+08	2.68
19,500	22,000	1,576	12.4	9.54	0.74	0.710	1.53	1.02E+09	0.580	2.99	8.60E+08	2.67
20,500	23,000	1,643	12.5	9.45	0.78	0.750	1.61	1.07E+09	0.600	2.95	9.02E+08	2.66
21,500	24,000	1,710	12.6	9.37	0.82	0.750	1.61	1.07E+09	0.610	2.86	9.43E+08	2.65
23,000	25,500	1,810	12.7	9.25	0.88	0.780	1.68	1.12E+09	0.640	2.80	1.00E+09	2.64
30,000	32,500	2,262	13.3	8.80	1.18	0.902	1.94	1.29E+09	0.782	2.62	1.29E+09	2.60
40,000	42,500	2,881	13.9	8.34	1.62	0.960	2.06	1.37E+09	0.900	2.26	1.68E+09	2.55

(a) $P_e = 8.42x10^4 A^{.93}$
(b) $V_{max} = P(3.1416)(.86)/TA$

Channel Length (L_c,(ft) =	6,000
Manning's n =	0.025
Ken + Kex =	1.1
Ocean Spring Amplitude (a_o,ft) =	2.15
Tidal Period (T,sec) =	44,640
Accel of Gravity (g,fps^2) =	32.2
Measured Tidal Prism (P) =	7.66E+08
Ratio Bay Tide to Ocean Tide =	0.33
Bay Amplitude (a_b) =	0.71
Tidal Prism through Existing Channel = 24400/39600 x P =	4.72E+08
Effective Bay Area (A_b) for Existing channel =	3.33E+08

$F = Ken + Kex +(2gn^2L_c/h_c^{4/3})$
Channel Width = $W_c =(A_c/p)^{1/(1+q)}$; where p = 1.164, q = 0.19
Cross-Sectional Area of Inlet = A_T = Channel Area + 2,500 sf (area of west channel)

performed for the cross-sectional area of the ebb channel, the stability curve is plotted using the cross-sectional area of the entire inlet. In this regard, the average cross-sectional area of the inlet west of the main ebb channel was added to each assumed channel cross-sectional area to obtain the total cross-sectional area of the inlet. The average cross-sectional area of the west side of Bogue Inlet is 2,500 square feet.

C.7. The sedimentary stability curve for Bogue Inlet was developed from the relationship between the tidal prism of an inlet and the equilibrium cross-sectional area as presented by Jarrett 1976. For inlets on the Atlantic Coast with a no jetties or a single jetty, the relationship between an inlet's cross-sectional area and its tidal prism is:

$$A = 5.37 \times 10^{-6} P^{1.07}$$

Rearranging this relationship, the tidal prism of an inlet can be computed from a known cross-sectional area by the following:

$$P = 8.42 \times 10^{4} A^{0.93}$$

Again, only the cross-sectional area of the ebb channel was used to determine the tidal prism applicable to the channel. The relationship between the tidal prism and the maximum velocity is:

$$V_{max} = P\pi C/TA$$

where: $C = 0.86$

Computations of V_{max} associated with the sedimentary stability of the inlet are also provided in Table C.1 with the resulting sedimentary stability curve superimposed on Figure C.4.

C.8. The sedimentary stability curve is seen to intersect the hydraulic stability curve at two points (A_c = 6,500 square feet and A_c = 33,500 square feet). These two cross-sectional areas both satisfy the hydraulic and sedimentary stability criteria, however, cross-sectional areas between 6,500 square feet and 11,500 square feet (critical area) are on the unstable side of the hydraulic stability curve indicating that the inlet would be considered marginally stable for areas in this range. The existing cross-sectional area of Bogue Inlet, which is 13,600 square feet, falls well within the stable region of the stability curve. Therefore, there is very little change that Bogue Inlet would ever close naturally.

C.9 The cross-sectional area of Bogue Inlet averaged over stations 10+00 to 50+00 (inlet throat section) that would be created immediately following the construction of the six channel alternatives and prior to the closure of the existing channel are considerably larger than the existing channel area averaged for the same stations (see Tables 5.1(a) and 5.1(b) in the main report). As a result, the stability analysis indicates that the cross-sectional area of the inlet throat will immediately begin to shoal and adjust toward its present cross-sectional area. For the case in which the existing channel is allowed to remain open, there would be some competition for the flow between the existing channel and the new channel. While the new channel should provide a more energy efficient path for waters exiting the sound, there is no guarantee that all of the shoaling necessary for the inlet throat to readjust to its present size will only occur in the existing channel. That is, there is still a chance that the new channel could shoal excessively given the reduced velocities associated with the two-channel system. For the case in which a dike is constructed across the existing channel, the effective cross-sectional area of the inlet would be immediately reduced to the values given in Tables 7.1(a) and 7.1(b) (Cross Sec. after shoaling) provided in the main report. All six of the channel alternatives satisfy the stability criteria, however, the cross-sectional area of the inlet with the 13.5-ft NGVD x 400 ft channel would be very close to the critical inlet area and would not provide a very large margin for error. Finally, given the uncertainty associated with the prediction of shoaling patterns for the case in which the existing channel is allowed to remain open following channel relocation, the stability analysis strongly supports the mechanical closure of the existing channel.

C.10. Horizontal Stability of New Channel. The preceding discussion focused on the ability of the new channel to capture the majority of the flow through Bogue Inlet and develop cross-sectional flow characteristics that would assure hydraulic stability of the channel. Hydraulic stability, however, is not related to the horizontal stability or migratory tendencies of the channel. In this regard, the existing inlet and associated channel is hydraulically stable but the channel has displayed a high degree of horizontal instability. The channel has undergone significant changes in position over the years with these changes including major shifts in position over a relatively short period of time and the more recent trend in which the channel has migrated rather steadily to the east. The artificial repositioning of the channel to a more central location between Bogue Banks and Bear Island will essentially emulate a major shift in the channel location similar to that which occurred during the mid 1970's. The artificial reconfiguration of the inlet may forego a future shift in the channel to this central location but will not prevent the relocated channel from migrating either to the east or to the west. While the channel did migrate slightly to the west following mid 1970's natural repositioning, the primary tendency of the channel has been to move toward the east. Based on this historic behavior, the artificially repositioned channel will also have a dominant tendency to migrate to the east.